Comparison of different capillary models to predict the hydraulic conductivity from the water retention curve.

J. Touma

IRD, UMR LISAH SupAgro-INRA-IRD, B.P. 434, 1004, Tunis, Tunisie
Phone: + 216 71 750 009, Fax: + 216 71 750 254, e-mail: touma@supagro.inra.fr

1. Abstract

The relative hydraulic conductivity relations of two contrasted soils, a clay and a sand, are predicted from their water retention expressed by the equation of van Genuchten in combination with three capillary models (Mualem, Burdine and Fatt & Dykstra) and used to compute infiltration amounts and rates according to the series Philip solution. The results are compared to the original results of Philip for the clay and to experimental data for the sand. For the clay, the results obtained with the first two capillary models are severely underestimated unlike those of the third one which are very close to the results of Philip. For the sand also, the results of the first two models are underestimated while those of the third one are very satisfactory. It is concluded that the conductivity curve predicted with capillary model of Fatt & Dykstra is reliable.

2. Introduction

The knowledge of the soil hydrologic properties is essential to study the water and solute movement in the vadose zone. These are the water retention, $\theta(h)$ and conductivity, $K(\theta)$, curves which relate the soil's water content, θ, to its water pressure head, h, and its hydraulic conductivity, K. While the determination of the water retention relation is generally easy, that of the conductivity curve is difficult, expensive and time consuming. This led to the development of predictive models to deduce the relative conductivity, K_r, from the retention curve. The relative conductivity is given by the ratio $K_r = K(\theta)/K_s(\theta_s)$, with subscript s referring to values at saturation. The various models differ by their representation of the pore space. Due to its flexibility, the equation of Van Genuchten (1980) is among the most used to describe the water retention curve. It is usually combined with the capillary model of Mualem (1976) to predict the soil water conductivity. We examine the combination of the equation of Van Genuchten with the capillary models of Burdine (1953) and Fatt & Dykstra (1951) in addition to that of Mualem. The resulting soil properties from the three combinations are used to compute the four terms series of Philip (1969) for the Yolo light clay and for the Grenoble sand. For the Yolo light clay, the results of each combination are compared to the original data and results of Philip, while for the Grenoble sand, they are compared to experimental data.

3. Material and methods

The equation proposed by Van Genuchten to describe the water retention relation is:

$$S = \left[1 + \left(h/h_0\right)^n\right]^{-m} \qquad (1)$$

In this equation S is the degree of saturation defined by: $S = (\theta - \theta_r)/(\theta_s - \theta_r)$, with θ_r the residual water content, h_0 (cm), n and m are parameters. Van Genuchten (1980) showed that, under the constraints $m = 1 - 1/n$ for the capillary model of Mualem and $m = 1 - 2/n$ for that of Burdine, the relative hydraulic conductivities can be expressed in closed-form. For Mualem it is given by:

$$K_r = \sqrt{S}\left[1 - \left(1 - S^{1/m}\right)^m\right]^2 \qquad (2)$$

And for Burdine, the expression is:

$$K_r = S^2\left[1 - \left(1 - S^{1/m}\right)^m\right] \qquad (3)$$

The combination of equation (1) with the capillary model of Fatt & Dykstra, cannot be expressed in a closed-form equation. However, with the constraint $m = 1 - 2.5/n$, an excellent approximation is obtained by:

$$K_r = S^{(2+2.5/mn)} \qquad (4)$$

Data for the Yolo light clay is taken from Philip (1969), and for the Grenoble sand from Touma & Vauclin (1986). The solid points in figures 1 and 2 show the data points of the Yolo light clay and the Grenoble sand respectively. Figures (1a) and (2a) refer to the water retention and (1b) and (2b) to the conductivity data. In figures (1a) and (2a) the dotted, dashed and solid lines are the fit of equation (1) on the water retention data points with the constraints of Mualem, Burdine and Fatt & Dykstra respectively; the resulting conductivities are shown in figures (1b) and (2b) by the dotted, dashed and solid lines respectively. Note that for both soils the conductivity curve according to each model is obtained from the corresponding predicted curve and the value of the saturated conductivity.

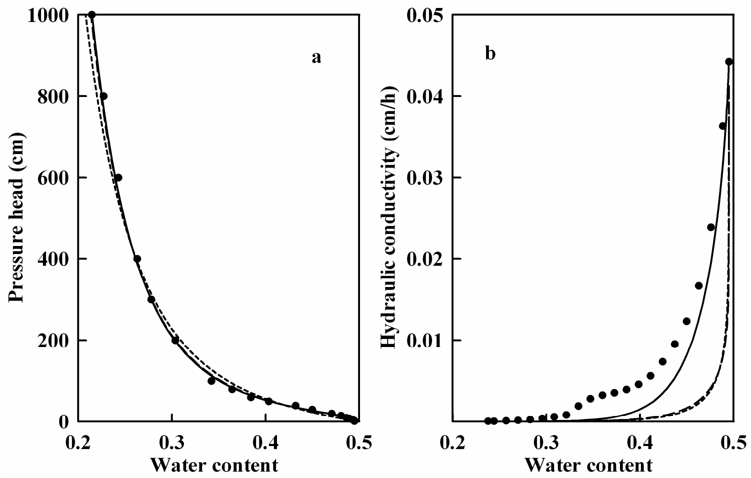

Figure 1 Comparison between data and soil properties obtained according to the three capillary models for the Yolo light clay

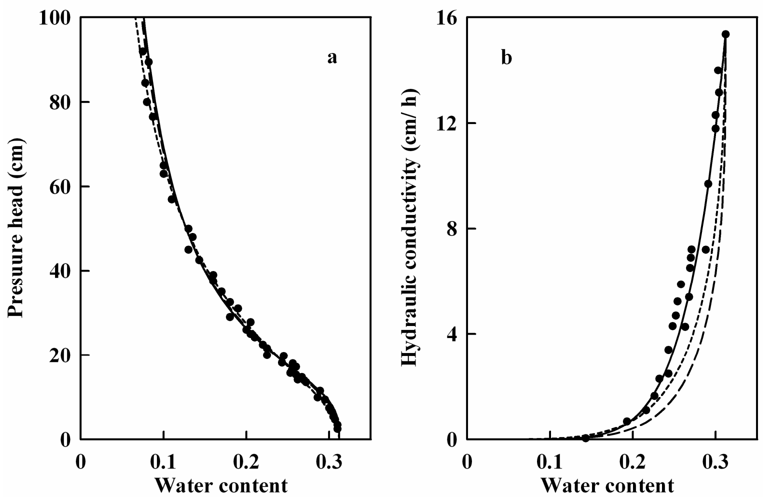

Figure 2 Comparison between data and soil properties obtained according to the three capillary models for the Grenoble sand

4. Results and discussion

First note that while the fitted retention curves with the three constraints are hardly distinguishable, the predicted conductivity curves are significantly different. For the Yolo light clay, the conductivities predicted by the capillary models of Mualem and Burdine severely underestimate the data. On the other hand, the predictions of Fatt & Dykstra are much closer to the observations especially for the higher water contents. Note that for this soil the saturation is at θ_s = 0.495. For the Grenoble sand the underestimation of the first two models is less pronounced, while the curve predicted by Fatt & Dykstra fits remarkably well the data.

In order to appreciate the quality of the prediction of each of the three capillary models, the four terms of the series solution of Philip (1969) denoted S_1,.., S_4, are evaluated with the properties predicted according to each one. For the Yolo light clay, Table 1 compares the resulting values to the original ones of Philip. The table shows also the value of t_{grav}, the time limit beyond which the series solution is not reliable and the profile at infinity must be applied Philip (1969). This table shows that the first two terms computed by the capillary models of Mualem and Burdine are less than half those of Philip. On the other hand, the values obtained with the model of Fatt & Dyksta are much closer to the reference values of Philip.

Table 1 Comparison of the computed four series terms $S_{1,..,}$ S4 and t_{grav} with those of Philip

Term	Philip	Mualem	Burdine	Fatt & Dykstra
S_1 (cm $\tilde{h}^{1/2}$)	0.7524	0.2774	0.3336	0.6469
S_2 (cm h^{-1})	$1.675\ 10^{-2}$	$5.431\ 10^{-3}$	$6.819\ 10^{-3}$	$1.903\ 10^{-2}$
S_3 (cm $h^{-3/2}$)	$3.035\ 10^{-4}$	$2.058\ 10^{-4}$	$1.730\ 10^{-4}$	$4.271\ 10^{-4}$
S_4 (cm h^{-2})	$1.161\ 10^{-6}$	$1.170\ 10^{-5}$	$4.672\ 10^{-6}$	$3.083\ 10^{-6}$
t_{grav} (h)	290	39	57	213

Figure 3 compares the infiltration amounts (3a) and rates (3b) respectively computed with the three capillary models with those of Philip up to t = 100h, which is practically twice t_{grav} of Mualem and Burdine and half that of Fatt & Dykstra. In this figure the points are the results of Philip, the dotted, dashed and continuous curves correspond to the capillary models of Mualem, Burdine and Fatt & Dykstra respectively. At t = 100h, the infiltration computed with the first two models is about half that of Philip. On the other hand, the difference between the infiltrated amount computed by the third model and that of Philip is less than 10%. Considering the infiltration rates on figure (3b), the profile at infinity for the models of Mualem and Burdine must be applied far before t_{grav} (at approximately $t = t_{grav}/3$ of the corresponding model), otherwise the infiltration rate would be less than Ks, which is physically irrelevant since theoretically this is the lower limit. By contrast, the infiltration rate computed with Fatt & Dykstra is very close to that of Philip.

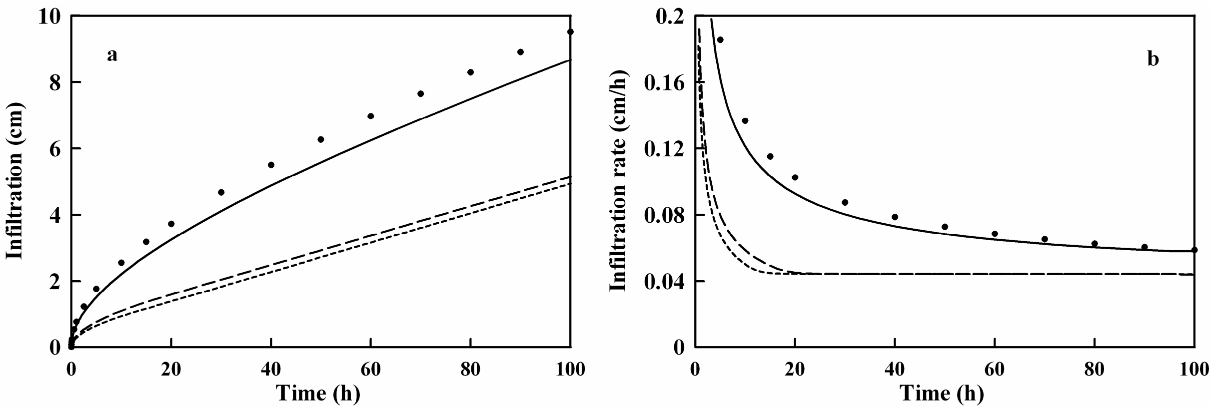

Figure 3 Comparison of the infiltration amounts (a) and rates (b) computed for the Yolo light clay by the three capillary models with those of Philip (1969)

For the Grenoble sand, the infiltration amounts and rates computed by the three capillary models are compared to experimental observations. The results are shown on figure (4a) and (4b) for the infiltration amounts and rates respectively. In these figures, the circles are data points, the dotted, dashed and continuous curves correspond to the capillary models of Mualem, Burdine and Fatt & Dykstra.

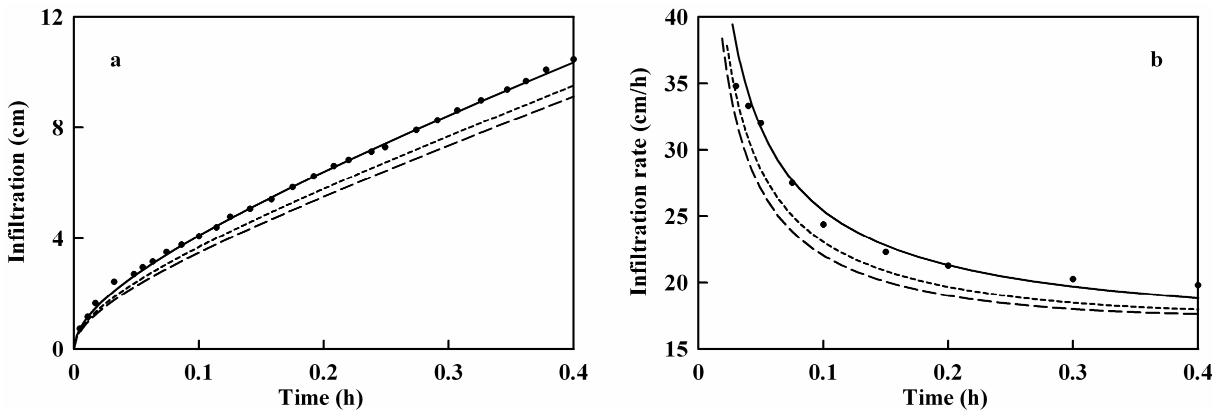

Figure 4 Comparison of the infiltration amounts (a) and rates (b) computed for the Grenoble sand by the three capillary models with experimental observations

As for the Yolo light clay, the results obtained by the first two model are underestimated to a lesser extent however, and without any anomaly. The results of the third model are prectically identical to observation.

Most probably the underestimation noticed with the first two models is due to the derivative of the conductivity with respect to the pressure head, which is infinite when the parameter n in equation (1) is less than 2 for Mualem and less than 3 for Burdine, which is frequently the case for fine graded soils. On the other hand, the conductivity resulting from the model of Fatt & Dykstra is well behaved and its derivative does not exhibit infinite values. Therefore, the capillary models of Mualem and Burdine give acceptable results for coarse graded soils, but they are not suited for fine graded ones, while the model of Fatt & Dykstra give satisfactory results for both types of soil.

In a discussion of several capillary models, Brutsaert (2000) showed that both models of Mualem and Burdine consider that the tortuosity of the soil depends solely on the size of the largest pores filled with water, while the model of Fatt & Dykstra considers that the tortuosity path through any pore depends not only on the size of the largest pores but also on all the pores filled with water up to that pore. This might explain the better agreement of this last model with observations.

5. References

Brutsaert, W. 2000. A concise parametrization of the hydraulic conductivity of unsaturated soils. Adv. in Water Resour., 23, 811-815.

Burdine, N.T. 1953. Relative permeability calculation from pore size distribution data. Trans. Am. Inst. Min. Eng., 198, 71-78.

Fatt, I. & Dykstra H., 1951. Relative permeability studies. Trans Am. Inst. Min. Eng., 192, 249-255.

Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res., 12, 513-522.

Philip, J.R. 1969. Theory of infiltration. In: Advances in Hydroscience, 5, Ven Te Chow (Editor), Academic Press, N.Y., pp. 215-296.

Touma, J. & Vauclin, M. 1986. Experimental and numerical analysis of two-phase infiltration in a partially saturated soil. Trans. Porous Media, 1, 27-55.

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