

# Rainfall-Sheet Flow Hydrodynamic Model on Sloping Land

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## 1. Abstract

Aiming at revealing the effect of rainfall splash on hydrodynamic features of sheet flow, with a simulated rainfall experiment in condition of changeable rainfall densities and changeable gradients. this text studied the raindrop kinetic energy, flow pattern and resistance coefficient of overland flow, established the rainfall-sheet flow hydrodynamic model.

## 2. Introduction

This work is a part of The National Research Project- Forest Affection on the Mechanism of Soil Erosion. Currently the study on rainfall runoff of Slope is according to the general flow of turbulent flow of the open channel. But is all of the rainfall runoff on slope belong to the turbulent flow ? In addition, the previous work on rainfall runoff of the slope, ignored affection of raindrops hitting power to the sheet flow .This way conceal the mechanical properties of actual rainfall runoff on slope, often got no real mechanism of soil erosion. Our study focuses on the hydraulic characteristics of rainfall runoff on slope, combining indoor artificial rain test, confirming the movement of the sheet flow patterns, deriving the rainfall-sheet model, determining the model parameters.

## 3. Methods

### 3.1 Test Equipment and Test Set

#### 1) Rainfall Device

We chose the Lateral Jet Rainfall Device of the State Key Laboratory of Soil Erosion and Dry-land Farming on Loess Plateau for the test. The man-made rainfall height is 16 m.

#### 2) Experiment Model

It is a self-made soil slot with adjustable slope (Fig-1). The slot is 2 m long, 0.5 m wide and 0.5 m deep,



Figure 1 Soil slot

### 3) Test Soil Samples

The test soil came from the Loess Plateau is riddled by the 10mm sieve. Dry bulk density is controlled in about 1.2 g/cm<sup>3</sup>,

### 3.2 Methods

#### 1) Rainfall Characteristics

##### ◆ Raindrop Diameter

The method to determine the size of raindrops is usual filter paper- splashing spot method.

##### ◆ Raindrop Landing Speed

We use a series of experience formula which is given by Mu Jin-ze to calculate raindrop terminal speed (m/s):

When 0.005cm<d<0.19cm:

$$V_m = 0.496 * k * 10^{\sqrt{28.32+6.524\lg(0.1d)-(\lg 0.1d)^2-3.665}} \quad (1)$$

$$\text{When } d < 0.15 \text{ cm, } K = 1.00 \quad \text{else } K = 1.00 - 0.53(d - 0.15),$$

When d=0.19~0.65cm:

$$V_m = (17.2 - 0.844 d)\sqrt{0.1d} \quad (2)$$

Where:  $V_m$ =raindrop terminal velocity, m / s

D=the diameter of raindrops, mm.

K=the deformation coefficients of diameter of raindrops

### 3.3 Hydraulic Model of Slope Flow

Generally the flat slope flow is regarded as one-dimensional, the assumption to the infiltration and rainfall is constant and uniform. So the slope flow is regarded as a steady flow (Fig-2).

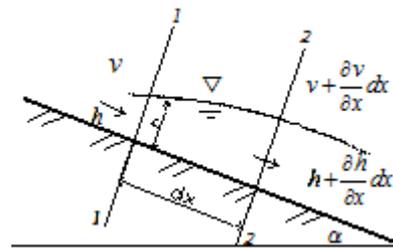


Figure 2 Slope flow unit diagram

#### 1) Slope Flow Continuous Equation

$$v \frac{\partial h}{\partial x} + h \frac{\partial v}{\partial x} = q \cos \alpha \quad (3)$$

Where: q= rainfall excess

V= average velocity of the cross section, Rainfall intensity minus penetration

h= water depth of the section.

#### 2) Slope Flow Momentum Equation

##### ◆ Pressure P

$$1\text{-}1\text{section } P_1: P_1 = \frac{1}{2} h^2 \cos \alpha \gamma \quad (4)$$

$$2-2 \text{ section } P_2: P_2 = \frac{1}{2} \left( h + \frac{\partial h}{\partial x} dx \right)^2 \cos \alpha \gamma \quad (5)$$

Where:  $\alpha$ =slope degree

$\gamma$ =water unit weight

$dx$ = distance 1-1 and 2-2 .

◆ **Water Unit Weight G to the Flow direction x**

$$G = \gamma \left( h + \frac{1}{2} \frac{\partial h}{\partial x} dx \right) dx \sin \alpha \quad (6)$$

◆ **Resistance T**

$$T = \tau \cdot dx = \gamma h J dx = 3 \mu v / h \cdot dx \quad (7)$$

Where:  $\mu$ = viscosity coefficient

◆ **Momentum  $M_1$  through 1-1 section flowing into water unit within dt**

$$M_1 = \rho \alpha_0 v^2 h dt \quad (8)$$

◆ **Momentum  $M_2$  of rainfall flowing into water unit within dt**

$$M_2 = \rho q dx \cos \alpha \sin \alpha v_m dt \quad (9)$$

◆ **Momentum  $M_3$  through 2-2 section flowing out of water unit within dt**

$$M_3 = \alpha_0 \rho \left( v + \frac{\partial v}{\partial x} dx \right)^2 \left( h + \frac{\partial h}{\partial x} dx \right) dt \quad (10)$$

Where:  $\alpha_0$ =momentum correcting coefficient.

So we have

$$\left\{ \begin{array}{l} - \left[ h \frac{\partial h}{\partial x} dx + \frac{1}{2} \left( \frac{\partial h}{\partial x} dx \right)^2 \right] \cos \alpha \gamma + \gamma \left( h + \frac{1}{2} \frac{\partial h}{\partial x} dx \right) dx \sin \alpha - 3 \mu v / h \cdot dx \\ = [\rho q dx \cos \alpha \sin \alpha v_m dt - \alpha_0 \rho \left( v^2 \frac{\partial h}{\partial x} dx + 2 v h \frac{\partial v}{\partial x} dx \right) dt] / dt \\ v \frac{\partial h}{\partial x} + h \frac{\partial v}{\partial x} = q \cos \alpha \end{array} \right. \quad (11)$$

Difference equation is:

$$h_2 = \frac{\gamma h_1 \sin \alpha - 3 \mu v_1 / h_1 - \rho q \cos \alpha \sin \alpha \cdot v_m + 2 \rho v \alpha_0 q \cos \alpha}{\gamma h_1 \cos \alpha + \rho v_1^2} \Delta x + h_1 \quad (12)$$

## 4. Results

### 4.1 Raindrop Size

$$D = 0.381 d^{0.7204} \quad R^2 = 0.953 \quad (13)$$

Where, D=actual raindrop diameter

d= paper- splashing spot diameter.

The raindrop diameters of test are the sizes of between 0.8-2.1mm.

### 4.2 Sheet flow pattern

As a small water depth on the slope, smaller than 0.2 mm in general, the Reynolds number (Re) is very

small, is in the range of 5 to 30. According to the criteria of open channel flow, all experimental flows are laminar flows.

### 4.3 Flow Model Test

#### (1) Known Factors of simulating Calculation

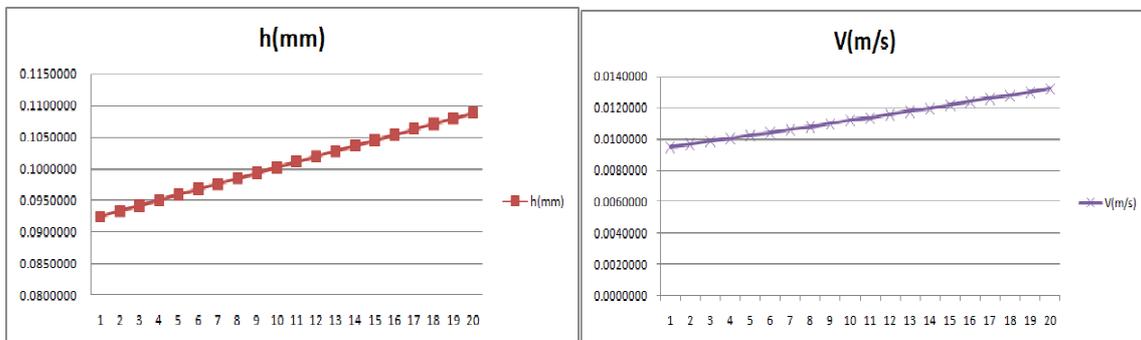
- rainfall excess  $q=30\text{mm/h}$ ; □ first section water depth  $h_1=0.093\text{mm}$ ; □ rainfall terminal velocity  $v_m=5.5\text{m/s}$ ;
- momentum correcting coefficient  $\alpha_0=1.10$ ; □ slope  $\alpha=15^\circ$ .

#### (2) Results of Simulating Calculation

See Table 1, Fig-3 and Fig-4:

**Table 1 Calculating flow data of the slot sections**

x(m)	0.5000000	1.0000000	1.5000000	2.0000000
h(mm)	0.0958600	0.1001800	0.1045100	0.1088300
V(m/s)	0.0102539	0.0111989	0.0121853	0.0132131
Q(cm <sup>3</sup> /s/m)	0.9829600	1.1219900	1.2735200	1.4380600



**Figure 3 Water depth changes along flow course**

**Figure 3 Water velocity changes along flow course**

#### (3) Comparisons between Simulating Calculation and Actual Test Flow

The differences of the water depth are 30%~50%. The differences of the water velocities are 10%~25%.

### 5. Reference

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